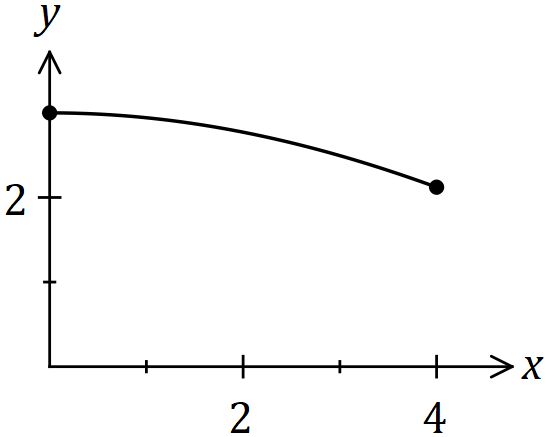
|  |  |
| --- | --- |
|  | **MATHEMATICS:SPECIALIST 3 & 4**  **SEMESTER 2 2019**  **TEST 4**  **Calculator Free** |
|  |  |

Reading Time: 2 minutes Time Allowed: 30 minutes

Total Marks: 26

Question 1 (4 marks)

The curve defined by , where , is shown below.

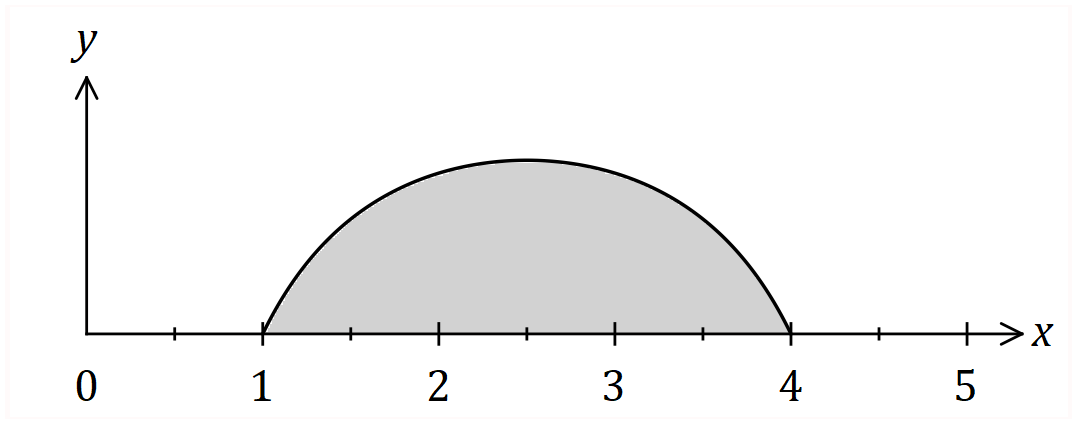


Determine the volume of the solid generated when the area bounded by the axis and the curve is rotated about the axis between and .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes integral   re-writes integral using double angle identity   integrates   substitutes both bounds and simplifies |

Question 2 (5 marks)

Part of the graph of is shown below.



Determine the shaded area, bounded by the curve and the -axis.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  integral, recognising need for partial fractions   obtains partial fractions   integrates   substitutes limits of integration   simplifies until just one logarithm remains |

Question 3 (5 marks)



**Question 4(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since    it follows that | |
| Mathematical behaviours | Marks |
| * identifies correct double angle formula to use * simplifies the integral to requiring the anti-derivative of * integrates correctly * evaluates the indefinite integral at the end points | 1  1  1  1 |

**Question 4(b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If we put    then we find that | |
| Mathematical behaviours | Marks |
| * calculates  correctly * substitutes into integral changing the limits appropriately * integrates the expression correctly * substitutes the boundary values and simplifies to a suitable form | 1  1  1  1 |

**Question 4(c) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| If we put  then  and the integral    Hence if the integral equals we conclude that *n* = 2018 | |
| Mathematical behaviours | Marks |
| identifies that   * identifies the most appropriate substitution * evaluates the integral correctly and thereby * deduces the correct value of | 1  1  1  1 |

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Question 6 (6 marks)

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Question 7 (4 marks)

|  |  |
| --- | --- |
| Solution | |
| If we write  Equating the coefficients gives  and  Hence | |
| Mathematical behaviours | Marks |
| * writes down the appropriate form of the partial fractions * compares coefficients to deduce the constants  and * integrates correctly * deduces the required result | 1  1  1  1 |

Question 8 (5 marks)

The region enclosed by the curves and , has an area of square units.

Determine the value of the positive constant .

|  |
| --- |
| **CAS solution** |
|  |

|  |
| --- |
| **Solution** |
| Intersect at and (CAS) |
| **Specific behaviours** |
| ✓ sketches curves   identifies points of intersection   correctly formed integral   evaluates integral in terms of   solves for |